

Linear Algebra 1

Charles Ndung'u

April 2023

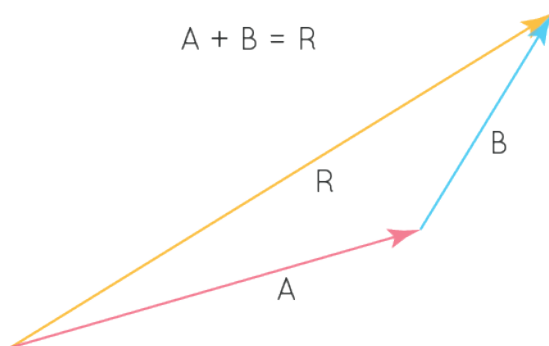
1 Introduction

Vectors and Scalars Vectors are physical quantities that have both magnitude and direction example displacement. Vectors are represented by a line segment with an arrow. The length of the line represent the magnitude of the vector while the arrow indicates the direction.

Types of vectors Resultant of two vectors

$$\vec{OA} + \vec{OB} = \vec{OC} : \hat{a} + \hat{b} = \hat{c}$$

Resultant Vector Representation



Example Find the resultant of the vectors $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $8\mathbf{i} + 6\mathbf{j}-10\mathbf{k}$.

Solution:

let our vector $\vec{A} + \vec{B} = \vec{C}$ adding the two is what will give us the resultant vector, thus $4\mathbf{i}+8\mathbf{i}=12\mathbf{i}$, $3\mathbf{j}+6\mathbf{j}=9\mathbf{j}$ and finally $4\mathbf{k}-10\mathbf{k}=-6\mathbf{k}$

we can conclude the resultant of the two vectors is $12\mathbf{i}+9\mathbf{j}-6\mathbf{k}$

Difference of two Vectors Here as the name difference states this means its the difference of two vectors.

Colinear Vectors

These are vectors that lie on the same line. Find if the following vectors are colinear: $\mathbf{p}(3,4,5)$ and $\mathbf{q}(6,8,10)$,

Solution: for two vectors to be colinear they must satisfy the following conditions: 1) Their cross product should be equal to 0.

2) The ratio of the corresponding coordinate are equal.

3) If there exist a scalar \vec{n} such that $\vec{p} = n.\vec{q}$

thus we will use the following conditions to determine if our vectors are colinear:

$$\frac{3}{6}=2$$

$$\frac{4}{8}=2$$

$$\frac{5}{10}=2$$

Thus they all have the same ratio meaning they are colinear.

Coplanar These are vectors that lie on the same plane or on parallel planes.

Are these vectors coplanar. $x=1,1,1$ and $y=1,3,1$ and $z=2,2,2$.

Note that they are coplanar if the tripple scalar product is 0 then they are coplanar. we shall solve this as follows.

$$x \cdot |y \times z|$$

First we will solve the cross product $|y \times z|$, then dot it with x.

$$\begin{array}{ccc} i & j & k \\ 1 & 3 & 1 \\ 2 & 2 & 2 \end{array}$$

thus $i(6-2)$, $j(2-2)$ and $k(2-6)$.

$i(4)$, $j(0)$, $k(-4)$. We then now dot the vectors,

$$1 \times (4i) = 4$$

$$1 \times (0) = 0$$

$$1 \times (-4k) = -4$$

**we add the results to get $4+0+-4=0$
thus the vectors are coplanar**

2

Unit Vectors

These are vectors that have a magnitude of 1 find the unit vector of $y=2i-4j+5k$ We first obtain the

magnitude, then $\frac{y}{|\mathbf{y}|}$ the magnitude of y $|\mathbf{y}|$ is given by $\sqrt{2^2 + -4^2 + 5^2}$

3

which gives 5 thus the unit vector is. $\frac{2i-4j+5k}{5}$
 which is the unit vector. getting the direction of the vector e.g in 2D $\sin\theta j = \frac{-4j}{5}, \cos\theta i = \frac{2i}{5}$

$$\sin 306.8699 + \cos 66.4218 = 1(1)$$